Variability in analytical and numerical values of collapse loads in structures

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Summary

Using numerical and analytical methods, collapse loads and reference stress solutions are derived for a plate containing a penny-shaped crack under tension and bending, pipe geometries under internal pressure containing axial and circumferential cracks external cracks and finally for a cracked C-ring specimen loaded in tension and a compact tension specimen. 3D Finite element calculations were performed, using the same material properties, for various valid boundary conditions and compared to, in some cases, to analytical solutions. It is shown that there is a variation of +/- 20% in the results for the reference stresses. The variation is dependent on both modelling boundary conditions, mesh size and quality, and difference in methods for analytical derivations. In most cases there is no valid reason why one method or boundary condition should be any more or less relevant to another. This suggests, especially in complex non-linear fracture mechanics analysis, that the uncertainty in the evaluation of the parameter should be taken into account and sensitivity analyses should be performed. When performing calculations in life assessment procedures of cracked components the variation due to calculating the parameters should also be considered using possibly probabilistic methods.

Introduction

High performance and parallel computing has allowed non-linear and time dependant calculations of 3D geometries to be modelled faster and more efficiently. In some cases calculations can now be performed interactively with the results of finite element calculations being incorporated into another code. Life assessment methods in the power generation industry use linear and non-linear fracture mechanics parameters such as $K$ and $C^*$ to predict initiation and crack growth of components at elevated temperatures. It is clear in these assessment methods that the correct evaluation of the relevant fracture mechanics parameters, for which the lifetime prediction times are dependent upon, will largely determine the accuracy of the life predictions. In this paper a general comparison of results is made using three Dimensional Finite Element analysis and analytical modelling of collapse loads. 3D meshes are used to model a plate under bending and tension, a pre-cracked pipes under internal pressure and bending with an axial elliptical crack, and a longitudinal pipe under pressure and 4-point bend containing a circumferential crack. Exact details and information is not provided for each method. But emphasis is place on the differences in results that are obtained from seemingly perfectly acceptable methods and boundary conditions. The differences are discussed in the light of life assessment methods for cracked components operating at elevated temperatures.
High temperature fracture mechanics

The arguments for correlating high temperature crack growth data essentially follow those of elastic-plastic fracture mechanics methods. For creeping situations [1-3] where elasticity dominates the stress intensity factor \( K \) may be sufficient to predict crack growth. However as creep is a non-linear time dependent mechanism even in situation where small scale creep may exist linear elasticity may not be the answer. By using the \( J \) definition to develop the fracture mechanics parameter \( C^* \) it is possible to correlate time-dependent crack growth using non-linear fracture mechanics concepts.

A simplified expression for stress dependence of creep is given by a power law equation which is often called the Norton’s creep law and is comparable to the power law hardening material giving:

\[
\varepsilon = A\sigma^N
\]

and by analogy for a creeping material

\[
\dot{\varepsilon} = C\sigma^n
\]

where \( A, C, N, n, \) are material constants \( \varepsilon, \dot{\varepsilon} \) and \( \sigma \) are the strain, creep strain rate and applied stress respectively. Equation 2 is used to characterise the steady state (secondary) creep stage where the hardening by dislocation interaction is balanced by recovery processes. The typical value for \( n \) is between 3 and 10 for most metals. When \( N=n \) for creep and plasticity the assumption is that the state of stress is characterised in the same manner for the two conditions. The stress fields characterised by \( K \) in elasticity will be modified to the stress field characterised by the \( J \) integral in plasticity in the region around the crack tip. In the case of large scale creep where stress and strain rate determine the crack tip field the \( C^* \) parameter is analogous to \( J \). The \( C^* \) integral has been widely accepted as the fracture mechanics parameter for this purpose [3].

The engineering method available to calculate \( C^* \) is one based on reference stress concepts [4]. Reference stress procedures are employed to evaluate \( C^* \) for feature and actual component tests where the load-line deformation rate is not available. By determining;

\[
C^* = \sigma_{ref} \dot{\varepsilon}_{ref} \left( \frac{K}{\sigma_{ref}} \right)^2
\]

where \( \dot{\varepsilon}_{ref} \) is the creep strain rate at the reference stress, \( \sigma_{ref} \) and \( K \) is the stress intensity factor. Usually it is most convenient to employ limit analysis to obtain \( \sigma_{ref} \) from

\[
\sigma_{ref} = \sigma_y \frac{P}{P_{lc}}
\]

where \( P_{lc} \) is the collapse load of a cracked body and \( \sigma_y \) is the yield stress. The value of \( P_{lc} \) will depend on the collapse mechanism assumed and whether plane stress or plane strain conditions apply. The collapse loads solutions are available for some geometries [5] and in some cases need to be derived numerically.
Figure 1: 3D Fe meshes for a) C-ring, b) Compact tension with side-groove, c) straight pipe with axial crack, d) bent pipe with circumferential crack, e) Plate under tension and bending, f) Plate mesh with penny-shape crack
The object of the present exercise is to highlight the differences in the calculated results from cracked components between numerical and analytical methods. Therefore only an outline of the Finite Element methodology is presented here. The ABAQUS Finite Element package was used in the FE analysis. Essentially in deriving \( \sigma_{\text{ref}} \) from equation 4 the collapse loads of the cracked geometries are needed. From equation 3 it is clear that the important factors that determine \( C^* \) for the component case is the correct evaluation of the material properties and an accurate calculation of the reference stress \( \sigma_{\text{ref}} \). In order to see the variability that exists for the different methods of calculating \( \sigma_{\text{ref}} \) a 3D finite element analysis was initiated. Figures (1,a-f) show examples of meshes that were employed. Considerable effort was placed in careful development of the meshes, using 20-noded brick elements. Since all the meshes contained cracks the crack front was modelled using collapsed elements. Since the runs were non-linear elastic-plastic and the meshes are modelled with cracks it was necessary to try various boundary conditions and loads in order to see the effect on the calculations.

The reference stress is a unique value of stress in a loaded body associated to a specific failure mechanism. It can be derived from the limit load of a component which is the value of the load parameter that corresponds to the end of the restricted plastic flow and to the initiation of the unrestricted plastic flow and collapse is the failure mechanism associated with the attainment of the limit load. The limit load is normally considered to be a global parameter and can be calculated numerically using a Load vs displacement curve as shown in figure 2a. However in cracked components the relevant parameter for limit load may not be global collapse but local collapse. The various life assessment codes suggest that \( \sigma_{\text{ref}} \) should be taken from collapse of the crack region [6,7]. Figure 2b shows a schematic example of the regions of local collapse around the crack tip that would constitute a relevant reference stress for a cracked body. The regions (1 and 2) do not necessarily correspond to global collapse (as shown in figure 2a). However they will give differences in the value for collapse loads which will in turn reflect on the reference stress calculations.

Figures 3-6 give a graphic summary of the variations that could be expected with the calculations of the collapse loads and \( \sigma_{\text{ref}} \) for the different geometries. Various comparisons are made between different FE solutions as well as analytical solutions and in the case of the C-ring specimen shown in Figure 3 the numerical and the analytical solution is compared with the actual experiment. Clearly the objective is to obtain consistent results across the board. This would then give confidence in the accuracy of the solution. However as can be observed from figures 3-6 this is not the case.

In deriving the values for the reference stresses in figures (3-6) care was taken to use the same mesh, material properties and boundary conditions for each set of data. In this way the variability of the results would only be attributable to the methodology of choosing the parameter and not to these other factors. Therefore it seems clear that depending on which criteria is used a different \( \sigma_{\text{ref}} \) can be calculated.

**Conclusions**

The present work which is not complete and is continuing highlights the variabilities found in deriving solutions for parameters, such as the reference stress \( \sigma_{\text{ref}} K \) and \( C^* \), relevant to fracture mechanics analysis of cracked components operating in the creep range. Using numerical and analytical methods, collapse loads and reference stress solutions were derived for a plate containing a penny-shaped crack under
tension and bending, pipe geometries under internal pressure containing axial and circumferential cracks external cracks and finally for a cracked C-ring specimen loaded in tension and a compact tension specimen. 3D Finite element calculations were performed, using the same material properties, for various valid boundary conditions and compared to in some cases to analytical solutions. It is shown that there is a variation of +/- 20% in the results for the reference stresses. The variation is dependent on both modelling boundary conditions, mesh size and quality, and difference in methods for analytical derivations. In most cases there is no valid reason why one method of calculation or boundary condition should be any more or less relevant to another. This suggests, especially in complex non-linear fracture mechanics analysis, that the uncertainty in the evaluation of the parameter should be taken into account and sensitivity analyses should be performed. Also performing life assessment calculations of cracked components, these differences due to methods of derivation of the parameters should be taken into account in terms of bounding the solutions.

References


Figure 2: a) Schematic diagram of the global displacement with respect to local collapse in a cracked component, b) Cross-section regions of collapse local to the crack that are considered
Figure 3: Comparison of collapse loads achieved for the C-ring specimen during the experiment with Finite Element and analytical predictions for 4 different materials [9].

Figure 4: Comparison of collapse loads using Finite Element and analytical predictions [8] for the plate

Figure 5: Comparison of collapse loads using Finite Element and analytical predictions for the different of pipe specimens under consideration

Figure 6: Comparison of reference stresses for CT specimens under different loads and the plate specimen using different analytical methods that are used in the codes.