Finite Element Simulation of Dynamic Crack Propagation Without Remeshing

ABSTRACT: Simulation of the crack growth for complex geometries is presented in this paper. Determination of the crack propagation direction under mixed mode conditions is one of the most important parameters in fracture mechanics. There are several criteria that have been developed to predict crack growth and its direction using linear elastic fracture mechanics (LEFM), many of which have recently been incorporated into finite element codes. These criteria are commonly adopted in the prediction of crack propagation in simple geometries and in straight crack paths. In more complex geometries, a more accurate determination of the crack propagation path, using remeshing methods can be employed. However, the remeshing technique usually suffers from the loss of strain energy density that can occur at the tip of the crack during the interpolation of field solutions. In this research work, the crack growth simulation is presented which allows for crack path deviation without the use of remeshing of the model. This method deals with a nonstraight crack growth path, is based on a node releasing technique and appropriate fracture criteria. The maximum principal stress and maximum strain energy release rate criteria is used in this paper exclusively. The results of simulation have been compared with experimental results as well as with numerical works of others that have been found in the recently published literature.

KEYWORDS: finite element method, crack propagation, node releasing, maximum energy release rate, strain energy density

Introduction

An important problem in linear elastic fracture mechanics (LEFM) is the prediction of crack growth and its direction in a component. Since the first introduction of LEFM theories in 1920’s substantial research in developing new theories and applications have been carried out. For example, several fatigue crack propagation and crack growth criteria under impact loading have been developed. Essentially, there have been two strategies for solving these problems. These are to

- Use an analytical approach for calculation of stress intensity factors (SIFs) as a function of crack length.
- Use general numerical methods such as the finite element method (FEM) and boundary element method (BEM) which can be applied for different boundary conditions and more complex geometries.

Using the FEM method the crack is denoted as a discontinuity displacement field that consists of two free surfaces during the separation. The change of mesh topology is one of the methods for discontinuity modeling. Many researchers have employed the mesh topology modification technique that uses the node release [1–4] element kill [5,6] and element split methods. Because of sudden variations due to stress gradient and complex stress distribution on the crack tip, the application of a fine mesh is necessary to be employed in order to achieve acceptable results. To prevent the excessive increase of computation time when using a fine mesh, the crack tip remeshing techniques have been introduced by Bittencourt et al. [3] and Bouchard et al. [4]. Moreover, the crack tip remeshing can help to modify distorted elements located right at the crack tip. However, for the cases with large deformation, the mesh distortion at the crack tip involves a loss of accuracy during the interpolation of field variables.

Other techniques can model crack discontinuity without using remeshing. Belytschko et al. [7] have introduced a meshless method where the discretisation is achieved by a model which consists of only
nodes. Rashid [8] has developed the arbitrary local mesh replacement method based on two distinct meshes. First, localized mesh moves with the crack tip and then fills the rest of the domain. Recently, the extended finite element method for modeling of discontinuity in mesh has been developed by other researches [9,10]. Sukumar and Prevost [10] have implemented a discontinuous function and the near-tip asymptotic functions which are added to the finite element approximation using the framework of partition of unity. This permits the crack to be represented without explicitly meshing the crack surfaces and crack propagation simulations can be conducted without the need for any remeshing.

Because of the recent development of general FEM codes, they are becoming more popular for modeling of crack growth for difficult geometries and complex mechanical conditions such as contact surfaces in addition to convenient output data manipulation during the advanced post processing. In this paper a program has been adapted to the general purpose finite element code (ABAQUS™) by reviewing the different criteria for determination of kinking angle, crack propagation by node release technique without using remeshing. In the following sections some examples have been implemented and the results have been compared with experimental results and numerical results of other published works.

Criteria Used in Crack Initiation and Growth

In order to extend the crack through the mesh in the modeled component during the FEM process, criteria for crack extension need to be in place concurrent with the FEM calculation. Furthermore if the direction of the crack extension needs to be determined the criteria for crack extension needs to be further improved to deal with this. Therefore for modeling of crack growth, in each time step three procedural items must be checked:

1. The critical loading condition for crack to initiate.
2. The direction that the crack is to propagate.
3. The extent to which the crack needs to grow.

For the evaluation of these criteria, the stress intensity factors (SIFs) and strain energy release rate needs to be calculated at every increment. For the prediction of crack initiation a critical value of SIFs is often used. Therefore it is possible to calculate values of SIFs for each time step and compare it to critical values of the material failure properties and if the values of SIFs are greater than critical values, the first condition is satisfied.

For an elastic-plastic material, crack tip opening displacement criterion is also used. This was originally introduced by Wells in 1961 [11]. In each time step, crack tip opening displacement value is calculated and compared with critical value. Also according to the Griffith and Irwin theory, crack initiation can be predicted by the calculation of the strain energy release rate \( G \). Potential energy of the elastic body is defined as below

\[
\Pi = U - F
\]

where \( U \) is the strain energy stored in body and \( F \) is the work done by external forces. \( G \) is the rate of potential energy variations with respect to crack area that has been shown in Eq 2.

\[
G = - \frac{\partial \Pi}{\partial A}
\]

According to this definition, Griffith criterion for crack growth is written as below

\[
\begin{align*}
G < G_f & \Rightarrow \delta_a = 0 \quad \text{no propagation} \\
G = G_f & \Rightarrow \delta_a > 0 \quad \text{propagation could start}
\end{align*}
\]

where \( G_f \) is critical strain energy release rate, and \( a \) is crack length and \( \delta_a \) is crack length variation [12].

Fortino and Bilotta [13] has introduced an algorithm for evaluation of crack growth extension in two-dimensional (2D) LEFM problems. The mentioned method is based on the energetic formulation of the coupled displacement-crack propagation problems. In addition, the \( G-R \) curve can be used to calculate

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\(^{3}\)Hibbitt, Karlsson and Sorensen Ltd. ABAQUS, version 6.3.1, 2003.
crack length. For example, in virtual finite element (VFE) method introduced by Gerken and Smith [14,15], the calculation of virtual crack length variation between two elements in each step time uses the G-R curve as a criterion.

For the determination of crack kinking angle, researchers have introduced several criteria. Some of these criteria determine the crack growth direction based on stresses and strains field at the crack tip. These criteria generally give acceptable results for LEFM, such as maximum principal stress, maximum circumferential stress [16], and maximum strain [17]. However, for (NLFM) more complicated methods are needed such as criteria that determine the crack growth direction based on energy distribution on a cracked body. The most commonly used criterion is the maximum strain energy release rate [18]. Some other criteria are based on the nature of crack creation such as criteria that use microvoid continuum damage for determination of crack growth direction. In these theories, the crack growth is controlled by the creation and propagation of microvoids in the vicinity of the crack tip. Therefore, the crack propagates in the direction that most of the voids have been nucleated [19,20].

In the next section criteria relating to maximum principal stresses, maximum circumferential stress, minimum strain density energy, and maximum strain energy release rate for crack growth direction are described in further detail. However, the maximum principal stress and maximum strain energy release rate criteria is used in this paper exclusively.

**Maximum Principal Stress Criterion**

According to this criterion crack propagates in the direction that is perpendicular to the maximum principal stress. To derive more accurate results from this method, the size of the mesh must be fine compared to other regions. Also for LEFM, it is better to use singular elements. Bouchard et al. [21] have studied the effect of mesh size and element type used in the crack tip. The accuracy of this method depends on the mesh size and for achieving accurate results, therefore, the application of remeshing is necessary. The approach that the present paper takes is described below:

1. A ring of elements around the crack tip is selected as shown in Fig. 1(a).
2. Stress tensor of each Gaussian point of ring elements is calculated by FEM.
3. Depending on the distance of crack tip from Gaussian point a weight function for stress tensor is considered ($w_i$).
4. Stress tensor of crack tip is calculated according to Gaussian point’s stress tensor and weight function of that point (Eq 4).
5. The directions of principal stresses of crack tip stress tensor and direction perpendicular to maximum principal stresses is calculated.

\[
[\sigma]_{\text{tip}} = \frac{\sum_{i=1}^{\text{Elements}} \sum_{j=1}^{\text{Int-P}} w_i [\sigma]_{ij}^M}{\sum_{i=1}^{\text{Int-P}} w_i} \quad (4)
\]

The results of this method are independent of the crack tip mesh structure and it gives the same results for meshes of different sizes. However, the method used by Bouchard et al. [21] is strongly depended on
the mesh structure and to get acceptable results remeshing is needed in their case. Acceptable results under a linear elastic condition have been achieved by the present method which implements four-node elements with four Gaussian points without considering the effect of singularity.

Maximum Circumferential Stress Criterion

This criterion which was introduced by Erdogan and Sih for elastic materials uses the maximum circumferential stress around the crack tip to allow propagation of the crack. Stresses on a circle with a radius centered at the crack tip in polar coordinates as shown in Fig. 2 are expressed as

\[
\sigma_r = \frac{1}{\sqrt{2\pi r}} \cos(\theta/2) \left\{ K_1[1 + \sin^2(\theta/2)] + \frac{3}{2} K_{II} \sin(\theta) - 2 K_{II} \tan(\theta/2) \right\}
\]

\[
\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos(\theta/2) \left\{ K_1 \cos^2(\theta/2) - \frac{3}{2} K_{II} \sin(\theta) \right\}
\]

\[
\tau_r = \frac{1}{\sqrt{2\pi r}} \cos(\theta/2) \left\{ K_1 \sin(\theta) + K_{II}[3 \cos(\theta) - 1] \right\}
\]

These equations are valid for both plane stress and plane strain cases. According to maximum circumferential stress criterion there is an angle (θ) that \(\sigma_\theta\) is maximum. To find maximum \(\sigma_\theta\), the derivative equation of \(\sigma_\theta\) is equated by zero.

\[
\frac{\partial \sigma_\theta}{\partial \theta} = \frac{1}{\sqrt{2\pi r}} \left\{ K_1 \sin(\theta) + K_{II}[3 \cos(\theta) - 1] \right\} = 0
\]

Solving Eq 6 gives the θ corresponding to the maximum \(\sigma_\theta\)

\[
\theta = 2 \arctan \left\{ \frac{1}{4} K_1 + \frac{1}{4} \sqrt{\frac{K_1^2}{K_{II}^2} + 8} \right\}
\]

The result that gives the sign as opposite to sign of \(K_{II}\) is the correct one. Using Eq 6 in mode I loading \((K_{II}=0)\), the crack propagation angle is zero. In mode II loading, by solving the equation \(K_{II}\) \(\left[3 \cos(\theta) - 1\right]=0\), the crack propagation angle is ±70.5°. So the maximum range of the crack propagation angle under LEFM is limited to an angle range of [−70.5° to 70.5°]. Figure 3 illustrates the plots of \(\sigma_\theta\) and \(\tau_r\) versus propagation angle for arbitrary \(K_1\) and \(K_{II}\). It can be seen that when the circumferential stress is maximum, the shear stress tends to become zero.

Minimum Strain Energy Density Criterion

This method which was introduced by Sih and MacDonald in 1974 suggests that the crack propagates in the direction in which strain energy density is minimum. Sih and MacDonald supposed that the factor
preventing crack growth is strain energy \((W_c)\). Therefore, crack extension is more probable to occur in the direction that the preventing factor is minimal. Strain energy density for those points ahead of the crack tip can be written as a function of angle \(\theta\) (Fig. 2) is [1]

\[
S(\theta) = a_{11}K_1^2 + 2a_{12}K_1K_{II} + a_{22}K_{II}^2
\]  

(8)

where the above factors \(a_{11}, a_{12},\) and \(a_{22}\) are defined as below

\[
a_{11} = \frac{1}{16\pi\mu}[(1 + \cos \theta)(k - \cos \theta)]
\]

\[
a_{12} = \frac{1}{16\pi\mu}[2 \cos \theta - (k - 1)]
\]

\[
a_{22} = \frac{1}{16\pi\mu}[(1 - \cos \theta)(k + 1) + (1 + \cos \theta)(3 \cos \theta - 1)].
\]  

(9)

To obtain the kinking angle of the crack, the minimum value of \(S(\theta)\) for different values of \(\theta\) have to be found.

**Maximum Strain Energy Release Rate Criterion**

Strain energy release rate \(G\) is the required energy to increase the length of the crack one unit ahead. The crack propagates in the direction that \(G\) is maximum. Therefore, crack propagation direction will be derived from the following equations:

\[
\begin{align*}
\left( \frac{dG}{d\theta} \right)_{\theta=\theta_0} &= 0 \\
\left( \frac{d^2G}{d\theta^2} \right)_{\theta=\theta_0} &\leq 0
\end{align*}
\]  

(10)

To use this criterion, first \(G\) is calculated by the use of \(J\) integral. This is a common method to calculate \(G\) in fracture mechanics. Under LEFM conditions, the \(J\) integral is equal to \(G\). \(J\) integral is introduced by Rice [24] as

\[
J = \int_{\Gamma} \left( Ud\gamma - t_i \frac{\partial u_i}{\partial \xi} d\xi \right)
\]

(11)

where \(U\) is strain energy density, \(t_i\) is the traction vector perpendicular to path \(\Gamma\), \(u_i\) is displacement vector, and \(d\xi\) is an element on the contour \(\Gamma\).

If no forces are applied on the crack surfaces, \(J\) value for quasistatic and isothermal condition is independent of path. Another method of calculating \(G\) is using a surface integral that is more accurate. de Lorenzi [25], derived a method to calculate \(G\) using the formulation of continuum mechanics of virtual crack extension. According to this method for 2D condition, \(G\) value is

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**FIG. 3—Typical circumferential and shear stresses around crack tip versus \(\theta\) angle in mixed mode.**
where $A$ is the surface between paths $\Gamma_0$, $\Gamma_1$ as shown in Fig. 4. $\Delta x_1$ is the virtual crack propagation value. This method is more accurate compared to contour integral computation especially for elastic-plastic conditions.

The procedure to calculate $J$ integral for a four-noded element with linear shape function according to the contour integral method is given in [26].

If $N_1, \ldots, N_4$ is considered as shape function for nodes of a four-noded element, then

$$
\begin{align*}
N_1 &= 1/4(1 - \xi)(1 - \eta) \\
N_2 &= 1/4(1 + \xi)(1 - \eta) \\
N_3 &= 1/4(1 + \xi)(1 + \eta) \\
N_4 &= 1/4(1 - \xi)(1 + \eta)
\end{align*}
$$

(13)

Jacobian matrix of this element is

$$
[Jac^{(e)}] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\
\frac{\partial y}{\partial \eta} & \frac{\partial x}{\partial \eta}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{r} \frac{\partial N_i^e}{\partial \xi} x_i^e & \sum_{i=1}^{r} \frac{\partial N_i^e}{\partial \eta} y_i^e \\
\sum_{i=1}^{r} \frac{\partial N_i^e}{\partial \eta} x_i^e & \sum_{i=1}^{r} \frac{\partial N_i^e}{\partial \xi} y_i^e
\end{bmatrix}
$$

(14)

$[Jac^{(e)}]^{-1} = \frac{1}{|J^{(e)}|} \begin{bmatrix}
\frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\
-\frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \xi}
\end{bmatrix}
$

(15)

Using these two equations, the Jacobian mapping of the isoparametric elements are defined for the area or line integration of the $J$ integral.

If $J_i^{(e)}$ is considered as the $J$ integral value for the part of the path that passes from element $i$ (Fig. 5),

FIG. 4—Calculation of $J$ using surface integral method.

$$
G = \frac{1}{\delta a} \int \int_A \left( \sigma_{ij} \frac{\partial u_i}{\partial x_j} - \omega \delta_{ij} \right) \frac{\partial \Delta x_1}{\partial x_j} dA
$$

(12)
then the total value of $J$ integral for whole path is

$$J^{(\text{Contour})} = \sum_{i=1}^{n} J_i^{(e)}$$

(16)

where $n$ is the number of elements that contour integral passes through.

To calculate $J_i^{(e)}$, the known values of stress, strain, and displacement of element Gaussian point, will be used. The $J_i^{(e)}$ value for both $\xi=$constant and $\eta=$constant values will be calculated and added together. The following equation shows the method of calculating $J_i^{(e)}$:

$$J_i^{(e)} = \sum_{q=1}^{\text{NGAUS}} I(\xi_p, \eta_q) W_q$$

(17)

where $J_i^{(e)}$ is for the part of path that $\xi=$constant and NGAUS is the number of the integration point on this path. $W_q$ is the weight factor related to $\eta_q$.

$$J_i^{(e)} = \sum_{p=1}^{\text{NGAUS}} I(\xi_p, \eta_q) W_p$$

(18)

where $J_i^{(e)}$ is for the part of the path that $\eta=$constant and NGAUS is the number of the integration point on this path. $W_p$ is the weight factor related to $\xi_p$.

$$J^{(e)} = J_i^{(e)} + J_i^{(e)}$$

(19)

The strain energy is

$$U = U_e + U_p$$

(20)

$$U_e = \frac{1}{2} \mathbf{\sigma}(\mathbf{e}) e \quad U_p = \int_{\Gamma_p} \bar{\mathbf{\sigma}} \bar{d}$$

(21)

where $U_e$ is the elastic strain energy and $U_p$ is the plastic strain energy.

The following equations give the traction vector perpendicular to path:

$$t_i = \left\{ \begin{array}{c} \sigma_{x(i)} n_1 + \sigma_{y(i)} n_2 \\ \sigma_{y(i)} n_1 + \sigma_{x(i)} n_2 \end{array} \right\}$$

(22)

$$n^T = [n_1, n_2] = \left[ \frac{E_1}{\sqrt{E_1^2 + E_2^2}} \frac{E_2}{\sqrt{E_1^2 + E_2^2}} \right]$$

(23)

$$E = \left( \begin{array}{c} E_1 \\ E_2 \end{array} \right) = \left\{ \begin{array}{c} \frac{\partial y}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \xi} \frac{\partial x}{\partial \eta} \\ \frac{\partial x}{\partial \eta} \frac{\partial x}{\partial \xi} - \frac{\partial y}{\partial \eta} \frac{\partial y}{\partial \xi} \end{array} \right\}$$

For $\eta = \eta_p$

(24)

$$dx = \frac{\partial x}{\partial \xi} d\xi, \quad dy = \frac{\partial x}{\partial \eta} d\eta$$

For $\eta = \eta_p$

(25)

$$ds = \sqrt{\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2} d\xi$$

For $\eta = \eta_p$

(26)

$$\frac{\partial (u,v)}{\partial x} = \sum_{i=1}^{n} \frac{\partial N_i^{(e)}}{\partial x} (u_i, v_i), \quad \frac{\partial (u,v)}{\partial y} = \sum_{i=1}^{n} \frac{\partial N_i^{(e)}}{\partial y} (u_i, v_i)$$

(27)

$$J_i^{(e)} = \int_{\Gamma_i} \left( U d\eta - t_i \frac{\partial u}{\partial x} d\xi \right) = \int_{-1}^{1} Id\xi$$

For $\eta = \eta_p$

(28)
FIG. 6—Geometry and coordinate systems for the branched crack [27].

\[
\int_{-1}^{1} \left[ \sum_{p=1}^{NGAUS} l(\xi_p, \eta_p) W_p = \sum_{p=1}^{NGAUS} \left\{ \frac{1}{2} \left[ \sigma_{xx}(e_{xx})_e + \sigma_{xy}(e_{xy})_e + \sigma_{yy}(e_{yy})_e \right] \frac{\partial y}{\partial \xi} + \bar{U}_p \frac{\partial y}{\partial \xi} \right\} W_p \right] \] (29)

For \( \xi = \xi_p \) equations are the same as before except for \( \partial \xi / \partial x \) and \( \partial y / \partial \xi \) should be replaced with \( \partial \xi / \partial \eta \) and \( \partial y / \partial \eta \).

If crack propagates self-similarly under mixed mode loading, stress intensity factor and strain energy release rate are related as following equations:

\[
G = \frac{\pi(k + 1)}{8\mu}(K_I (1 + K_{II} \sin \alpha)) \] (30)

\[
\begin{align*}
k = 3 - 4\nu \quad &\text{For plane strain} \\
k = \frac{3 - \nu}{1 + \nu} \quad &\text{For plane stress}
\end{align*}
\] (31)

where \( \nu \) is the Poisson ratio, \( \mu \) is the shear modulus of material and \( K_I, K_{II} \) are the stress intensity factors.

When a part is under mixed mode loading, kinking angle is not zero, so Eq 30 does not stand. Nuismer [27], has developed the stress intensity factor in a branched crack tip with angle \( \alpha \) as shown in Fig. 6

\[
K_I = \frac{1}{2} \cos \frac{\alpha}{2} [K_I (1 + \cos \alpha) - 3K_{II} \sin \alpha] \] (32)

\[
K_{II} = \frac{1}{2} \cos \frac{\alpha}{2} [K_I \sin \alpha + K_{II} (3 \cos \alpha - 1)] \] (33)

Therefore, the \( G \) value for the present crack tip is

\[
\bar{G}_{(0)} = \frac{\pi(k + 1)}{8\mu}(K_I^2 + K_{II}^2) \] (34)

Substituting \( K_I \) and \( K_{II} \) in Eq 34 gives

\[
\bar{G}_{(0)} = \frac{\pi(k + 1)}{32\mu} \cos^2 \theta \left[ \frac{K_I^2}{2} (1 + \cos \theta)^2 + 9K_{II}^2 \sin^2 \theta - 6K_I K_{II} \sin \theta (1 + \cos \theta) \right] \] (35)

According to the maximum strain energy release rate criterion, crack propagates in the direction that \( \bar{G}_{(0)} \) is maximum. To find the maximum \( \bar{G}_{(0)} \), the derivative of \( \bar{G}_{(0)} \) is equated to zero (\( \partial \bar{G}_{(0)} / \partial \theta = 0 \)). This equation gives the crack propagation angle

\[
\cos(\theta) = \frac{3K_{II}^2 + K_I \sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2} \] (36)
Equation 36 is equivalent to Eq 7. On the other hand, under LEFM conditions, maximum circumferential stress criterion and maximum strain energy release rate criterion are the same. When crack is not self-similar, the maximum strain energy release rate under mixed mode loading can be defined by substituting Eq 36 in Eq 35.

\[
\tilde{G}_{\theta, \max} = \frac{\pi (k + 1) K_1^4 + 24 K_1^4 + 12 K_1^2 K_1^2 + K_1^2 (K_1^2 + 8 K_2^2)^{3/2}}{16 \mu (K_1^2 + 9 K_2^2)}
\]  

(37)

**Implementation of Automatic Node Release in ABAQUS**

User subroutines are used in the ABAQUS code to introduce new and nonconventional required numerical conditions. In crack propagation problems, because of dynamic boundary condition updates, it is necessary to change the boundary condition (crack surfaces) during each solving time step. Therefore, node release technique implemented into the multipoint constraint subroutine (MPC) has been used. In this method instead of conventional meshing that uses one node at each element corner, four nodes are located at an identical coordinates. Figure 7 shows this mesh. Nodes a, b, c, and d of four elements A, B, C, and D are coupled together. This means that their displacements are forced to be the same.

\[ u_a = u_b = u_c = u_d \]  

(38)

To create a crack surface, it is necessary for these nodes to be separated following the crack growth direction. Consequently equal displacement of remaining attached nodes is estimated. Figure 8 shows three different possible situations that the direction of a crack can take.

If for a point that contains nodes a, b, c, and d the constraint equation can be written as

\[
k_1^1 u_a + k_2^1 u_b + k_3^1 u_c + k_4^1 u_d = 0
\]

\[
k_1^2 u_b + k_2^2 u_c + k_3^2 u_d = 0
\]

\[
k_1^3 u_c + k_3^3 u_d = 0
\]

(39)

The above-mathematical representation of multipoint constraint for nodes a, b, c, and d can be

**FIG. 7**—2D mesh with multinode construction.

**FIG. 8**—Representation of three possible node release situation for creation of crack surfaces: (a) Crack extension is straight \( u_a = u_c \) and \( u_b = u_d \); (b) crack extension is right direction \( u_a = u_b = u_c \); (c) crack extension is left direction \( u_a = u_b = u_d \).
reexpressed in the ABAQUS input file format as

\*MPC
\a,\a,\b,\c,\d
\b,\b,\c,\d
\c,\c,\d

According to crack propagation direction, the factors in Eq 39 will take the values of 0, 1, or −1. Table 1 shows the factors in Eq 39 for three different situations, shown in Fig. 8.

This strategy can only be used to model three possible situations of crack propagation, which differs approximately by 90° from each other. Any crack propagation angle less than 90° between each case is extrapolated to one of the above cases. This mean that some localized error in the evaluation of the crack growth direction can reduce the accuracy in short term. In Ref. [2], a method to solve this problem has been introduced. This is shown in Fig. 9. The nearest node ahead of the crack growth direction path can be moved in a way that one of the element faces is aligned in the new crack direction.

The method that has been used in this paper is as follows:

1. Amount of crack propagation in each step is calculated by the program or defined by user (\(\Delta \alpha\)).
2. According to crack propagation angle and \(\Delta \alpha\) value, the new candidate of crack tip point will be defined (\(t'\)).
3. The nodes around the \(tt'\) line will be released according to the given algorithm.

The element size used in this method should be at least five times smaller than \(\Delta \alpha\). So in each step of crack propagation modeling, more than one node can be released. Therefore, it is possible to model every crack propagation angle with a good degree of approximations by releasing the nodes around propagation line. Figure 11 shows the algorithm of crack propagation simulation.

<table>
<thead>
<tr>
<th>Crack extension is left handed</th>
<th>Crack extension is right handed</th>
<th>Crack extension is straight</th>
</tr>
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<tbody>
<tr>
<td>(k_1^1) = (k_1^2) = 1</td>
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<td>(k_1^1) = (k_1^2) = 1</td>
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<tr>
<td>(k_1^2) = (-1)</td>
<td>(k_1^2) = (-1)</td>
<td>(k_1^2) = (-1)</td>
</tr>
<tr>
<td>(k_1^3) = (k_1^2) = 0</td>
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</tbody>
</table>

FIG. 9—Automatic remeshing strategy for crack propagation: (a) previous mesh; (b) current; mesh [2].

FIG. 10—Method of approximation of crack trajectory with node release technique.
Numerical Examples

To model crack propagation using maximum principal stress criterion and maximum strain energy release rate criterion, different programs have been written in VISUAL-FORTRAN language and linked to the ABAQUS 6.3.1 code. This program has been implemented for four-noded linear elements with four Gaussian points successfully. The examples have been selected from Refs. [3,4] to compare the results with those that used remeshing techniques.

Example 1: A Rectangular Block with an Oblique Precrack

Figure 12 shows the geometrical properties of a plate with the rectangular shape that contains an oblique precrack. The dimensions are in millimeter. The material is purely elastic with a Young modulus of $E = 98000$ MPa and a Poisson ratio of $\nu = 0.3$.

In Fig. 13 the mesh for the rectangular plate has been shown and the plane strain condition was used with four-noded elements containing four integration points. The element type used in this analysis is CPE4. A total number of 20,975 elements and 83,900 nodes have been used in this example. The element
size around crack tip is about 40 μm. The crack propagation length in each step has been numerically tested for a range of 4–15 elements and similar results were obtained. Therefore, a length of seven elements was selected arbitrarily. Figure 14 shows crack propagation trajectory according to the maximum principal stress criterion. To use the maximum strain energy release rate criterion, it is necessary to calculate $J$ integral and stress intensity factors around the crack tip. Figure 15 shows $G$ values for example 1 around the crack tip during crack propagation.

Figure 16 shows the results of the crack propagation path modeling using maximum strain energy release rate criterion for the rectangular part with an oblique crack.

Figure 17 illustrates the comparison of the result from the present work with the remeshing technique results published in [4].
Example 2: Drilled Plate with Two Supporting Point

The geometrical properties and loading condition of example 2 are shown in Fig. 18. The material of this part is polymethyl methacrylate (PMMA). It has a preliminary notch with a defined length in a particular distance from centerline. The experimental results of this model have been studied at Cornell University and published in Ref. [3]. The experimental results have also been compared with a finite element modeling result using the remeshing technique. Elastic modulus of material used were \( E = 2.94 \) GPa and the Poisson ratio is \( \nu = 0.3 \) [28]. Because of the holes in the model acting as stress concentrators, the shape of the crack propagation path is seen to be as a curvature. Figure 19 shows the mesh of this model. The element type is the same as example 1 but because of the boundary condition, the plane stress element has been used (CPS4 from the ABAQUS library). A total number of 17 657 elements and 70 628 nodes have been used in this example and the element size around the crack tip is 0.1524 mm.

Figure 20 shows the crack propagation path using maximum principal stress criterion for this model. Figure 21 compares the result from this model using maximum principal stress and maximum strain energy release rate criteria with experimental results and also results from Ref. [3].

FIG. 15—\( J \) integral histories for example 1.

FIG. 16—Crack trajectory based on maximum strain energy release rate criterion.

FIG. 17—Comparison of crack trajectory based on maximum principal stress and maximum strain energy release rate criteria with results of remeshing.
Conclusions

The simulation of the crack growth for complex geometries using LEFM methods is presented in this paper. A program which deals with assessing the stress/strain distribution local to the crack tip in 2D has
been developed to link with a general purpose finite element program. This development allows for the modeling of complex geometries in the engineering applications. The maximum principal stress and maximum strain energy release rate criteria have been used to find the extent of crack growth, its direction, and length. Since the remeshing technique usually suffers from the loss of strain energy density that can occur at the tip of the crack during the interpolation of field solutions, a regular quad mesh with no remeshing has been adopted in this research work. It has been shown that the presented method can conveniently deal with a nonstraight crack growth path, which is based on a node releasing technique. The results of the simulation have been favorably compared with published experimental and numerical results from the literature. The presented results showed a good agreement with the published experimental data and results from numerical simulations that employed remeshing techniques. Since the present technique is sufficiently robust and adaptable, it is also suggested that this method can be used to simulate crack growth for more ductile materials that undergo elastic-plastic and creep deformation.

References

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